

به نام او

آمار و کاربرد

۹۵/۹/۳۰

رگرسیون-۲

$Y \quad \hat{Y}$

$$S_x = \frac{1}{n-1} \sum (x_i - \bar{x}_i)$$

$$\sigma^r \rightarrow s^r = \frac{1}{n-r} \sum_{i=1}^n (Y_i - (a + bx_i))^r$$

$$b \sim N\left(\beta, \frac{s^r}{\sum x_i^r}\right) \rightarrow \frac{b - \beta}{\sqrt{\frac{s^r}{\sum x_i^r}}} \sim T_{n-r}$$

$$\frac{s}{\sqrt{\sum x_i^r}} = SE \quad \frac{b - \beta}{SE} \sim T_{n-r}$$

$$P\left[b \in \beta \pm \frac{s}{\sqrt{\sum x_i^r}} t_{n-r, 0.95}\right] = 0.90$$

$$\Leftrightarrow P\left[b \pm \frac{s}{\sqrt{\sum x_i^r}} t_{n-r, \alpha} \supseteq \beta\right] = 1 - \alpha$$

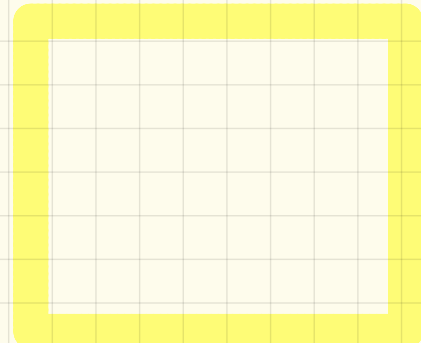
$$t_{n-1, \gamma} = \frac{b}{SE} \rightarrow \gamma: P\text{-value}$$

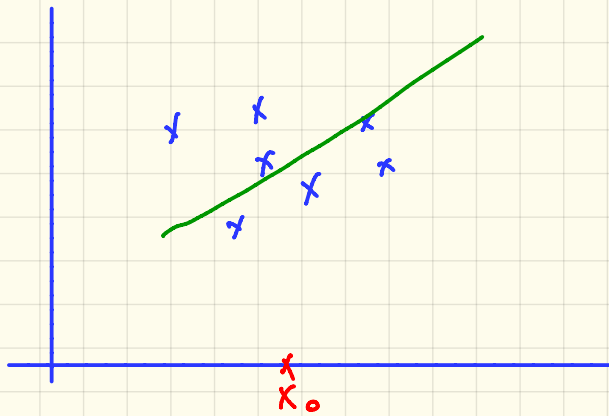
$$\hat{b} \rightarrow \gamma_1: P\text{-value (right)}$$

$$\hat{b} \rightarrow 1 - \gamma_2: P\text{-value (right)}$$



$$t_{n-1, \gamma} = \frac{b}{SE} \quad \hat{b} \quad \frac{1 - \gamma_2}{\gamma_1} \rightarrow \text{right P-value}$$





α, β

a, b

$$a = N \left(\alpha, \frac{\sigma^2}{\sum x_i^2 + \frac{1}{n}} \right)$$

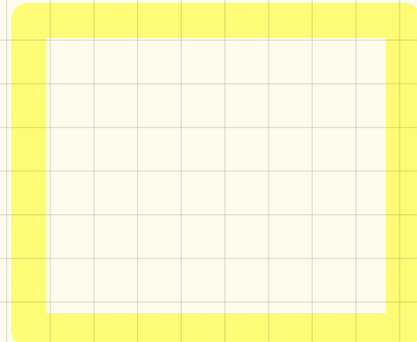
$$x_0 \rightarrow a + bx_0 \rightarrow Y_0 = \alpha + \beta x_0 + e_0$$

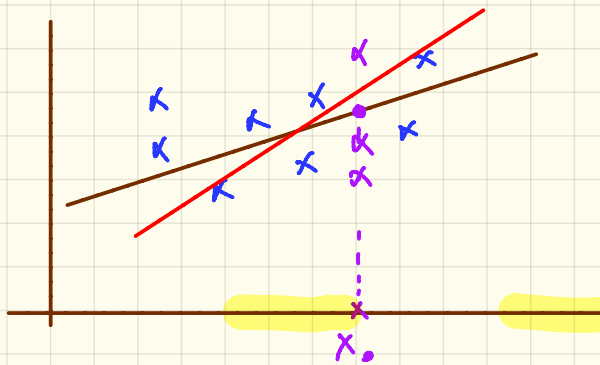
$$E[Y|X_0] = \alpha + \beta x_0$$

$$\text{var}(Y_0) = \text{var}(\alpha + \beta x_0 + e) = \text{var}(e)$$

$$\text{var}(\alpha + \beta x_0 + e_0) = \text{var}(\alpha + \beta x_0) + \text{var}(e_0)$$

\downarrow
 واریانس e_0





 $\alpha + \beta X$
 $a + bX$

$$a + bX_0 \rightarrow \alpha + \beta X_0 = \mu_0$$

$$\rightarrow \alpha + \beta X_0 + e_0$$

$$\mu_0 = a + bX_0 \pm t_{0.1} \cdot \sigma \sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum x_i^2}} \rightarrow a + bX_0 = N\left(\mu_0, \sigma^2 \left(\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum x_i^2}\right)\right)$$

$$Y_0 = a + bX_0 \pm t_{0.1} \cdot \sigma \sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum x_i^2} + 1}$$

$$N\left(\mu_0, \sigma^2 \left(\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum x_i^2} + 1\right)\right)$$

